ART. IX.—Fuller's Calculating Slide-Rule.

By James J. Fenton.

[Read 10th September, 1885.]

SLIDE-RULES, for use in approximate calculations, are not nearly so well known as they deserve to be; for, whilst possessing all the advantages to be derived from logarithms, they are entirely divested of their attendant technicalities. If a book of logarithms be placed in the hands of any intelligent person, unskilled in mathematics-no matter how well the method of using them, and their great advantages over the ordinary methods have been explained—it is most unlikely he will take the trouble to master them; but with the slide-rule it is very different, for, as the logarithms themselves are entirely ignored, and ordinary numbers alone are dealt with, he might, by the aid of a few simple rules, in a very short time become quite proficient in manipulating it. With the rule it is not necessary, as in the case of tabular logarithms, to look first for the numbers, then for the corresponding logarithms, adding thereto the differences for the last figure, transcribing them to paper, finding their sum or difference (as the case may be), and then reversing the process, so as to translate the result into ordinary notation. These operations, simple in themselves, often take so long that many expert calculators can (except in calculations involving the powers or roots of numbers) in most cases obtain the result in less time. In the case of the slide-rule, however, all these obstacles are avoided, and an ordinary result in multiplication, division, squares, or square-roots may be obtained at once by inspection with one or two simple movements of the scale, the mental operations of addition or subtraction being mechanically performed by the rule itself.

The logarithmic scale, in its simplest form, consists of a rule or line divided into parts proportional to the logarithms of the natural numbers from 10 to 100. Take a line of any length and assume it to be made up of 1000 equal parts; then mark off the number of such divisions corresponding to the logarithms (indices being omitted) of 20, 30, 40, &c., to 100, viz., 301, 477, 602, &c., to 1000; and place opposite to these the corresponding numbers, after which the scale

may be completed by interposing in like manner the intermediate numbers, and any others in the third place that the length of scale will allow. Now, if with a pair of compasses the space spanned from the commencement of the scale to any number be simply added on to some other number, the resulting product will be found indicated at the lower leg of the compass, and similarly if the space between two numbers be measured, and an equal space be laid off from the top of the scale, the resulting quotient will be also shown at the lower point. A very few experiments with this scale, however, will exhibit one defect, viz., that the lower leg of the compass often falls altogether out of the scale; and hence the necessity of a double scale for practical utility, such as is adopted in the common Carpenter's Slide-Rule.* In this rule, which is usually about a foot long, the results are obtained by placing scale against scale. There are, in fact, four distinct scales, marked A, B, C, and D respectively—the three former, which are alike, being double logarithmic scales, and the fourth being a single logarithmic scale exactly double the length of the others. This latter is employed for finding, in conjunction with the other scales, squares and square-roots. This slide-rule is well known, and is, I believe, much used in England amongst carpenters, mechanics, and others for rough calculations. But in this colony I have not met with a single workman who understood its use, and it is usually looked upon merely as a useful adjunct to the foot-rule for the measurement of inches.

Another form of logarithmic rule to which reference ought to be made is the circular one, which, although not so well known as the common slide-rule, possesses many advantages over it. In the circular rule only a single scale is necessary; the slide is dispensed with, and the operations are performed by two hands or indices instead.+1

* An interesting account of the history and use of this Rule is to be found in a pamphlet entitled "The Carpenter's Slide-Rule," published by

Messrs. John Rabone and Sons, of Birmingham.

[†] Since the reading of this paper, I have had an opportunity of examining a most ingenious and portable form of the circular logarithmic scale under the name of the "Cercle à Calcul." In size and general appearance this instrument resembles a watch. It has two hands or indices, one fixed and the other movable, so that they may be placed in any required relation to each other; and on the two faces are engraved the scales. One of these faces is movable by means of a thumb-screw, such as is used in a keyless watch, whilst the opposite face is fixed, and may be traversed by a needle on the same pivot as, and with corresponding motions to, the needle (or movable index) on the

The rules just referred to, however, will only give results correct to two figures, and on this account they have been available only for rough approximate calculations, or merely looked upon as mathematical curiosities. To give results with even three figures one would require (reckoning twenty divisions to the inch) a straight rule 8 ft. 4 in. long, or a circular rule 1 ft. 4 in.; and to give results with four figures, one 83 ft. 4 in. or 13 ft. 3 in. respectively; and hence their inapplicability for other than approximate calculations.

This great difficulty in regard to length of scale has, however, at length been overcome by arranging the scale on a spiral. The instrument I exhibit to-night—the Calculating Slide-Rule of Professor George Fuller, C.E., of Queen's College, Belfast, Ireland—has a single logarithmic scale, 500 inches in length, wound in a spiral form round a cylinder barely six inches long by three inches in diameter. The old plan of placing scale against scale has been abandoned, and two indices—one fixed and the other movable—are substituted instead. This rule will give correct results to four and sometimes to five figures, and is therefore much more reliable than a table of four-figure logarithms. There is, moreover, an additional scale for finding the logarithms themselves if required, and on the inner cylinder of the slide are arranged, for ready reference, many useful mathematical tables and formulæ, including a table of natural sines.

In regard to matters of calculation generally, I may state that I have received the greatest assistance in statistical calculations from the Arithmometer, Logarithms, and Reciprocals. In the calculation of percentages, or in calculations involving a constant divisor, I consider Reciprocals* by far the most convenient and readiest of the three methods just named. The Arithmometer, I am of opinion, is still unsurpassed when exact results in over six figures are

* A valuable work on Reciprocals is by Lieut, Col. Oakes, A.I.A. London:

C. & E. Layton, 1865.

opposite side. The movable face has three scales—viz., a scale of numbers, a scale of squares, and a scale of sines; whilst the fixed face contains a scale of cubes, and a scale for finding, in conjunction with the scale of numbers, the logarithm of a number, or vice versa. By means of these scales, ordinary results in multiplication and division may be obtained; the squares, cubes, square and cube roots may be found simply by inspection, and rough arithmetical and trigonometrical calculations involving such powers and roots may be readily made,—correct to the second, and often with an approximation to the third, figure. In many respects the "Cercle" is much more useful and convenient than the Carpenter's Slide-Rule; its price in France is 30 francs (about 25s.). A less portable but more useful instrument, having a greater length of scale, is also obtainable.

required; also in the calculation of logarithmic or other series. But the Spiral Slide-Rule decidedly supersedes the use of all three when results involving not more than four figures are required. And when it is considered that few calculations—at all events actuarial or statistical ones—can be carried, to any purpose, beyond the fourth or fifth figure, chiefly on account of the unreliability of the data, the universal utility of this rule will be at once recognised. Take, for example, the calculation of a death-rate based on the population of a country. No one would surely imagine that the number showing the population is correct in the unit's or ten's place, and even the figures in the hundred's and thousand's place can seldom be relied on. In deducing a result of any value, it would therefore be necessary that the number of figures in the result (quotient) should not exceed the number of reliable figures in the divisor (or population); in fact, it ought to be one less.

ADDENDUM.

[Written 6th November, 1885.]

An objection often raised to the use of Slide-Rules generally is the trouble experienced (although there is no doubt that in the great majority of cases it may be done simply by inspection) in finding out where to place the decimal point in the result; but in the Spiral Slide-Rule, by a simple device,

this difficulty has been overcome.

In calculating with the Spiral Slide-Rule it is advisable that the operations should be so arranged that the result may always be found at the *fixed* index. In using "constant" multipliers or divisors (as in the calculation of percentages, &c.), moreover, it will be found advantageous to set the "constant" once for all the operations in which it may be required. In the following examples of multiplication and division let C be constant:—

(1.) Multiplication with a Constant Factor. $C \times d = x, C \times d' = x', C \times d'' = x'', &c., may be resolved into$ $\binom{1}{C} = \frac{d}{x} = \frac{d'}{x'} = \frac{d''}{x''} = &c.,$

i.e. $(\log I - \log C) = \log d - \log x = \log d' - \log x' = \&c.$

(2.) Division with a Constant Divisor.

$$\frac{d}{C} = x$$
, $\frac{d'}{C} = x'$, $\frac{d''}{C} = x''$, &c., may be resolved into

$$\binom{C}{I} = \frac{d}{x} = \frac{d'}{x'} = \frac{d''}{x''} = \&c.,$$

i.e.
$$(\log, C - \log, 1) = \log, d - \log, x = \log, d' - \log, x' = \&c.$$

This resolves the operations into questions of proportion. If, therefore, in the first example, the indices be so arranged that I on the scale is opposite the *movable* index, and C opposite the *fixed* index, the ratio $\frac{I}{C}$ will be represented, logarithmetically, by the distance between the two indices; and as this ratio is the same for all the other ratios, viz., $\frac{d}{x}$, $\frac{d'}{x'}$, $\frac{d''}{x'}$, &c., it will be only necessary to bring the various values, d, d', d'', &c., to the *movable* index, and the results, viz., x, x', x'', &c., may be read off, in turn, at the *fixed* index. In like manner, in the second example, the *fixed* index is set at I, and the *movable* index at C; and then the scale is moved so as to bring the different values—d, d', d'', &c.—to the *movable* index, when the answers—x, x', x'', &c.—will be found at the *fixed* index.*

ART. X .- Note on the Habits of Hermit Crabs.

By A. H. S. Lucas, M.A., B.Sc.

[Read 12th November, 1885.]

A STATEMENT is constantly repeated in the text-books of zoology that the hermit crabs always protect their soft abdomens by taking up their abode in the *empty* shells of gasteropods. Thus Nicholson says: "The animal is compelled to protect the defenceless part of the body in some

^{*} It is stated (in a footnote) in the Instructions issued with the Rule that the two stops, which were fixed to the instruments first made, so that the beginning of the scale (100) might be brought at once to the fixed index, are now omitted as useless; but this is to be regretted, as, from the second set of examples shown above, it will be seen that such stops will prove of great advantage.